Fourier Analysis on some unimodular Lie groups

Teacher
Alessandro Palmieri

Course description
In this course, we consider the extension of the notion of Fourier transform for two classes of unimodular Lie groups: 1) compact Lie groups, 2) nilpotent Lie groups. The key object to introduce the definition of Group Fourier Transform $\mathbb{G}$ is the unitary dual $\hat{\mathbb{G}}$ which consists of the equivalence classes of irreducible, strongly continuous and unitary representations up to intertwining operators.

1) In the compact case, by Peter-Weyl theorem it follows that any element in $\hat{\mathbb{G}}$ is finite dimensional and one finds that the Plancherel measure in this case is a discrete measure. We consider some results that generalize the ones for the classical Fourier series on the torus.
2) In the nilpotent case, we restrict ourself to the example provided by the Heisenberg group H_n, which is the simplest example of not commutative stratified Lie group. By Stone-Von Neumann theorem it is possible to represent $\hat{\mathrm{H}}_n$ via the so-called Schrödinger representation $\{\pi_\lambda\}_{\lambda \in \mathbb{R}^*}$. By using these representations it is possible to recover the Plancherel formula and the inversion formula.

For the attendance of the course, a basic knowledge of Fourier transform (in the Euclidean case), Fourier series and Lie groups is required.

Course period
January-June 2025

SSD
MATH-03/A

Course References
[1] Fischer V., Ruzhansky M., *Quantization on nilpotent Lie group* (2016)
[2] Folland G.B., *A Course in Abstract Harmonic Analysis* (1995)
[3] Ruzhansky M., Turunen V., *Pseudo-Differential Operators and Symmetries. Background Analysis and Advanced Topics* (2009)
[4] Stein E.M., *Topics in harmonic analysis, related to the Littlewood-Paley theory* (1970)
[5] Thangavelu S., *Harmonic Analysis on the Heisenberg Group* (1998)

Credits and Hours
2 credits of lectures (16 hours)

Exam Modality
Seminar about one of the topics discussed during the course (the topic must be approved by the teacher before the exam).

Personal data

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Current Position

February 2022 – January 2025: Assistant Professor at the Mathematical Department of the University of Bari (tenure-track position), SSD MAT/05 (Mathematical Analysis)

Former positions and education

August 2021 – January 2022: Post-doc fellowship, Tohoku University, Sendai, Japan. JSPS Postdoctoral Fellowship for Research in Japan (Short-term). Advisor: Prof. H. Takamura (Tohoku University);

February 2019 – January 2020: Post-doc fellowship, University of Pisa, Pisa, Italy. (Project PRA 2018 49). Advisors: Prof. V. Georgiev (University of Pisa) and Prof. V. Magnani (University of Pisa);

October 2015 – September 2018: Ph.D. student at the Technical University Bergakademie Freiberg, Freiberg, Germany, Faculty of Mathematics and Computer Science, Institute of Applied Analysis, supported by Sächsiches Landesgraduirtenstipendium. Supervisors: Prof. M. Reissig (TU Freiberg), Prof. M. D'Abbicco (University of Bari).

Italian national scientific qualification for Associate Professor

2018-2020 national scientific qualification for Associate Professor - sector 01/A3 (Mathematical Analysis, Probability and Mathematical Statistics) from 07/05/21 to 07/05/32

Visiting Periods

August 2023: Zhejiang Normal University, Jinhua, China and Fudan University, Shanghai, China (invited by Prof. N.-A. Lai and Prof. Y. Zhou);

February 2023: Tohoku University, Sendai, Japan (invited by Prof. H. Takamura);

October 2018 – December 2018: Tohoku University, Sendai, Japan (invited by Prof. H. Takamura);

February 2017: Friedrich-Schiller-Universität, Jena, Germany (invited by Prof. W. Sickel).

Membership

ISAAC (*International Society for Analysis, its Applications and Computation*): ISAAC member since 30/04/18 and life member since 29/07/19;

INdAM (*Istituto Nazionale di Alta Matematica*): INdAM member since 2017 in the GNAMPA group (*Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni*).

Grants

2024: Co-investigator of GNAMPA's project "Modelli locali e non-locali con perturbazioni non-lineari", coordinator Dr. Giovanni Girardi (Università Politecnica delle Marche);

2024: Principal investigator of the ERC Seeds UNIBA project "Nonlinear Wave Equations in Not Euclidean Settings";

2022: Co-investigator of GNAMPA's project "Equazioni dispersive e dissipative: stime e profili asintotici", coordinator Prof. Marcello D'Abbicco (Università di Bari);

2020: Co-investigator of GNAMPA's project "Problemi stazionari e di evoluzione nelle equazioni di campo nonlineari dispersive", coordinator Prof. Jacopo Bellazzini (Università di Sassari/Università di Pisa);

2017: Co-investigator of GNAMPA's project "Equazioni di tipo dispersivo e proprieta asintotiche", coordinator Prof. Marcello D'Abbicco (Università di Bari).

Talks as Invited Speaker at Conferences and Workshops (since 2022)

29/01/2024: Fourier Analysis and Partial Differential Equations II, University of Ferrara, Ferrara, Italy, title of the talk: *On a nonlinear wave equation in the contracting de Sitter spacetime*;

27/08/2023: Workshop on Nonlinear Dispersive Equations, Fudan University, Shanghai, China, title of the talk: *A Nakao-type weakly coupled system with nonlinearities of derivative-type*;

20/07/2023: 14th ISAAC Congress, University of São Paolo, Ribeirão Preto, Brazil, title of the talk: *On a blow-up result with critical nonlinearities for a wave equation in the expanding de Sitter spacetime*;

26/04/2023: Workshop "PDE days at TU Bergakademie Freiberg" TU Freiberg, Freiberg, Germany, title of the talk: *Some results for semilinear wave equations on compact Lie groups*;

21/02/2023: The 24th Northeastern Symposium on Mathematical Analysis, Tohoku University, Sendai, Japan, title of the talk: *Blow-up results for a semilinear wave equation in the expanding de Sitter spacetime*;

29/11/2022: Conference on Noncommutative Analysis and PDEs, Queen Mary University of London, London, England, title of the talk: *Semilinear wave models on compact Lie groups*.

Organization of Workshops and Sessions

19/12/2023: Co-organizer with Profs. M. D'Abbicco, A. Loiudice and S. Lucente of the workshop "EDP e Dintorni: VIII Meeting around PDE", University of Bari, Bari, Italy;

17-21/07/2023: Co-organizer with Profs. M. D'Abbicco and M. Reissig of the Session 13 "Recent Progress in Evolution Equations" of the 14th ISAAC Congress, University of São Paolo, Ribeirão Preto, Brazil;

21/12/2022: Co-organizer with Profs. M. D'Abbicco, A. Loiudice and S. Lucente of the workshop "EDP e Dintorni: VII Meeting around PDE", University of Bari, Bari, Italy;

November 2021 - January 2022: Co-organizer with Dr. W. Chen of the series of webinars "Recent Advances in Nonlinear Evolution Equations" (RANEE);

6-8/02/2020: Co-organizer with Prof. V. Georgiev of the workshop "Dispersive equations of Math Physics", University of Pisa, Pisa, Italy.

Research database profiles
MATHSCINET: MR Author ID 1225290
Scopus: Scopus ID 57195262104
Web of Science: Web of Science ResearcherID AAR-5762-2020
Teaching activity
a.y. 2023/2024 - today: course *Analisi Matematica I* at UniBa, Department of Physics, bachelor degree in Physics;

a.y. 2023/2024 - today: course *Matematica I* at UniBa, Department of Physics, bachelor degree in Material Science and Technology;

a.y. 2022/2023: course *Fourier Analysis and its Applications* at UniBa, Ph.D. Programme in Computer Science and Mathematics (XXXVIII cycle);

a.y. 2022/2023 - today: course *Matematica* at UniBa, Department Earth and Geo-environmental Sciences, bachelor degree in Geological Sciences;

a.y. 2021/2022 - 2022/2023: course *Analisi Matematica* at UniBa, Department of Computer Science, bachelor degree in Computer Science and Digital Communication;

a.y. 2016/2017- 2017/2018: Teaching Assistant for the course *Method of analysis and Econometrics* at TUBA Freiberg, Faculty 6.

Papers in scientific journals (since 2020)

1. W. Chen, R. Ikehata, A. Palmieri, Asymptotic behaviors for Blackstock's model of thermoviscous flow, *Indiana Univ. Math. J.* **72**, 2 (2023)

2. A. Palmieri, H. Takamura, A blow-up result for a Nakao-type weakly coupled system with nonlinearities of derivative-type, *Math. Ann.* **387** (2023), 111-132

3. A. Palmieri, Lifespan estimates for local solutions to the semilinear wave equation in Einstein-de Sitter spacetime, *Appl. Anal.* **102**(13) (2023), 3577–3608

4. A. Palmieri, H. Takamura, On a semilinear wave equation in anti-de Sitter spacetime: the critical case, *J. Math. Phys.* **63**, 111505 (2022)

5. A. Palmieri, H. Takamura, A note on blow-up results for semilinear wave equations in de Sitter and anti-de Sitter spacetimes, *J. Math. Anal. Appl.* **514**, 126266 (2022)

6. A. Palmieri, A global existence result for a semilinear wave equation with lower order terms on compact Lie groups, *J. Fourier Anal. Appl.* **28**, 21 (2022)

7. M. Hamouda, M.A. Hamza, A. Palmieri, Blow-up and lifespan estimates for a damped wave equation in the Einstein-de Sitter spacetime with nonlinearity of derivative type, *Nonlinear Differ. Equ. Appl.* **29**, 19 (2022)

8. A. Palmieri, Semilinear wave equation on compact Lie groups, *J. Pseudo-Differ. Oper. Appl.* **43**, 12 (2021)

9. M. Hamouda, M.A. Hamza, A. Palmieri, A note on the nonexistence of global solutions to the semilinear wave equation with nonlinearity of derivative-type in the generalized Einstein-de Sitter spacetime, *Commun. Pure Appl. Anal.* **20**(11) (2021), 3703–3721

10. A. Palmieri, Integral representation formulae for the solution of a wave equation with time-dependent damping and mass in the scale-invariant case, *Math. Meth. Appl. Sci.* **44**(17) (2021), 13008–13039

11. W. Chen, S. Lucente, A. Palmieri, Nonexistence of global solutions for generalized Tricomi equations with combined nonlinearity, *Nonlinear Anal. RWA* **61**, 103354 (2021)

12. S. Lucente, A. Palmieri, A blow-up result for a generalized Tricomi equation with nonlinearity of derivative type, *Milan J. Math.* **89** (2021), 45–57

13. A. Palmieri, Z. Tu, A blow-up result for a semilinear wave equation with scale-invariant damping and mass and nonlinearity of derivative type, *Calc. Var.* **60**, 72 (2021)

14. A. Palmieri, Blow - up results for semilinear damped wave equations in Einstein - de Sitter spacetime, *Z. Angew. Math. Phys.* **72**, 64 (2021), doi:10.1007/s00033-021-01494-x

15. A. Palmieri, On the blow – up of solutions to semilinear damped wave equations with power nonlinearity in compact Lie groups, *J. Differential Equations* **281** (2021), 85–104

16. V. Georgiev, A. Palmieri, Lifespan estimates for local in time solutions to the semilinear heat equation on the Heisenberg group, *Ann. Mat. Pura Appl.* **200** (2021), 999–1032

17. M. D'Abbicco, A. Palmieri, A note on $L^p - L^q$ estimates for semilinear critical dissipative Klein–Gordon equations, *J. Dyn. Diff. Equat.* **33** (2021), 63–74

18. W. Chen, A. Palmieri, A blow-up result for the semilinear Moore-Gibson-Thompson equation with nonlinearity of derivative type in the conservative case, *Evol. Equ. Control Theory* **10**(4) (2021), 673–687

19. A. Palmieri, H. Takamura, Nonexistence of global solutions for a weakly coupled system of semilinear damped wave equations in the scattering case with mixed nonlinear terms, *Nonlinear Differ. Equ. Appl.* **27**, 58 (2020)

20. A. Palmieri, Decay estimates for the linear damped wave equation on the Heisenberg, *J. Funct. Anal.* **279** (2020) 108721

21. W. Chen, A. Palmieri, Nonexistence of global solutions for the semilinear Moore- Gibson-Thompson equation with power nonlinearity in the conservative case, *Discrete & Continuous Dynamical Systems - A* **40**(9) (2020), 5513–5540

22. A. Palmieri, A note on a conjecture for the critical curve of a weakly coupled system of semilinear wave equations with scale-invariant lower order terms, *Math. Meth. Appl. Sci.* **43** (2020), 6702–6731

23. V. Georgiev, A. Palmieri, Critical exponent of Fujita-type for the semilinear damped wave equation on the Heisenberg group with power nonlinearity, *J. Differential Equations* **269**(1) (2020), 420–448

24. A. Palmieri, H. Takamura, Nonexistence of global solutions for a weakly coupled system of semilinear damped wave equations of derivative type in the scattering case, *Mediterr. J. Math.* **17**, 13 (2020)